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Absorption of sound by homogeneous turbulence

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The following problem is treated: given a plane acoustic wave propagating through an unbounded field of turbulence, calculate the amount of acoustic energy converted into turbulent kinetic energy. The fluid velocities due to the acoustic waves and the turbulence are assumed to be small compared with the speed of sound. Thus the sound–turbulence interaction is weak and the turbulent field may be considered to be incompressible. The analysis is based on the interaction of two opposite effects: the acoustic distortion of the turbulence (producing anisotropic Reynolds stresses) and the redistribution of the kinetic energy among components (tendency towards isotropy) and among wavenumbers (energy cascade and dissipation). These phenomena are described using semi-empirical turbulence arguments. It is seen that the simplest model for the redistribution among components is not sufficient for unsteady flows. A more complete model is used which is modified to agree with the exact instantaneous distortion analysis of Ribner & Tucker to first order. Owing to the two redistribution effects, the Reynolds stress behaves inelastically and is out of phase with the acoustic field. Thus there is an average production of turbulent energy corresponding to the absorption of acoustic energy and attenuation of the incident wave. For nearly isotropic turbulence, the attenuation coefficient is found to be proportional to the rate of viscous dissipation and independent of the frequency.

In order to compare the theory with experiment several constants involved in the semi-empirical model of the turbulence must be found. Owing to the lack of better information these constants are estimated here by order-of-magnitude considerations. No existing experiments correspond to the homogeneous turbulence assumed by the theory. Comparison with the few reasonably applicable experiments shows qualitative agreement though the importance of the turbulent absorption is generally of nearly the same order as the measurement error. Several discrepancies between jet noise experiments and aerodynamic noise predictions may be roughly explained using the above analysis.

1. Introduction

A plane wave propagating through a homogeneous gas at rest attenuates owing to molecular relaxation, heat conduction and viscous friction. The attenuation coefficient (relative energy loss per unit length) for viscous and heat-conduction effects is proportional to the square of the frequency of the incoming wave. Molecular-relaxation effects are particularly important for oxygen and nitrogen in air and are dependent on

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water-vapour concentration (Evans, Bass & Sutherland 1972). These effects also often increase with frequency. Temperature and velocity gradients involve local variations of the speed of sound, and give rise to refraction and scattering. These effects lead to changes in the distribution of sound pressure in space without changing the total acoustic energy. Most atmospheric absorption measurements include scattering and refraction as parasitic effects. Refraction is strongest for acoustic wavelengths small compared with the scale of the gradients. A strong attenuation of a beam of sound can be caused by interactions with turbulent velocity and temperature fluctuations, involving scattering of the incident wave (Brown & Clifford 1976). Although scattering does not involve a change in the total acoustic energy, it gives rise to a scattered wave (rotated wavenumber vector) whose energy propagates out of the original beam. The 'attenuation coefficient' for such a case is found to be dependent on the frequency of the incident wave. Another scattering-based analysis, by Howe (1973), also finds a redistribution but no net change in acoustic energy.

A number of experimental measurements of the absorption of sound in the atmosphere are available, but they are generally unreliable owing to the unknown variations in refraction and ground effects. For instance, Wiener & Keast (1959) measured an excess attenuation in hilltop-to-hilltop transmission of the order of 10^{-3} to $10^{-4}\,\mathrm{m}^{-1}$ which was independent of the frequency, and attributed it to turbulence. Measurements by Dneprovskaya, Iofe & Levitar (1963) and those discussed by Brown & Clifford (1976) give similar and larger attenuations. However, to a large extent these attenuations can be attributed to beam spreading due to scattering.

Hunter & Lowson (1974) made the first experimental attempt to evaluate the attenuation coefficient for a wave propagating through turbulence, excluding non-turbulent effects and turbulent scattering. They measured an attenuation coefficient of the order of $10^{-3}\,\mathrm{m}^{-1}$ which was independent of the frequency for all measured frequencies (600–5000 Hz). This experiment will be discussed in more detail in §5. This experiment indicates that another attenuation mechanism can occur which is not strongly dependent on the frequency. The question arises whether this effect is due to the absorption of acoustic energy by turbulence. This kind of interaction could become much more important in an intense turbulent flow, like a jet. One of the conclusions of this work is that turbulent absorption as well as emission and refraction of sound should be considered when dealing with jet noise.

The present analysis depends upon certain assumptions regarding the temporal evolution of turbulence. As the acoustic time scales are generally much shorter than the turbulent time scales, most investigators have assumed that there is no change in the turbulence structure during the interaction. From a mathematical point of view, this enabled them to consider turbulence to be unchanged by the sound wave and considerably simplified the analysis in this limit; one then speaks of *frozen* turbulence. In the present study, the absorption of sound by turbulence is investigated with the Reynolds stresses allowed to fluctuate and to evolve with time under the action of the sound wave; thus we shall model *non-frozen* turbulence.

The present analysis is related to that of Crow (1967, 1968), who introduced 'memory functions' to describe the 'viscoelastic' properties of fine-grained isotropic turbulence. However, the present analysis models the mechanisms involved in the energy transfer at high Reynolds numbers.

As a first attempt to solve the problem of the absorption of sound by a frozen pattern of turbulence, Noir (1975) and George (1973) used a modal analysis similar to that of Chu & Kovasznay (1958). The sound wave amplitude was characterized by a small parameter δ and the turbulent velocity fluctuations by a small parameter ϵ. The resulting absorption coefficient was found to be of order $\epsilon^2\delta^2$, which is several orders of magnitude smaller than that suggested by measurements.

In this paper we present a method which deals with non-frozen turbulence by modelling the unsteady behaviour of the Reynolds stress as coupled to the sound wave.

2. The Reynolds-stress equation

From an energy point of view, the absorption of acoustic energy by turbulence produces additional turbulent kinetic energy which is eventually dissipated by viscosity. The production term in the turbulent kinetic energy equation involves mean-flow gradients and the normal Reynolds stresses. A model of the coupling between the Reynolds stress and the mean flow will be seen to result in a time-average production of turbulent kinetic energy.

The incident sound field (mean flow) is given by a plane harmonic wave

$$U_1 = \delta\gamma^{-1}a\cos(kx_1-\omega t),$$

where δ is a small dimensionless parameter characteristic of the wave amplitude, γ is the adiabatic exponent, a the speed of sound, k the acoustic wavenumber and ω the corresponding angular frequency; the sound field is compressible and time dependent. The undisturbed turbulence is assumed to be homogeneous and turbulent density fluctuations are neglected. The equation of conservation of mass is

$$\frac{\partial\rho}{\partial t}+U_i\frac{\partial\rho}{\partial x_i}+\rho\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

for the mean flow and

$$u_i\frac{\partial\rho}{\partial x_i}+\rho\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

for the turbulent field, where ρ and U_i are the mean density and velocity fields and u_i is the fluctuating velocity field. The Reynolds-stress equation is

$$\begin{aligned}\frac{d}{dt}\rho\overline{u_iu_j}+\frac{\partial}{\partial x_k}[\rho\overline{u_iu_ju_k}+\overline{p(u_i\delta_{jk}+u_j\delta_{ik})}-(\overline{u_i\sigma_{jk}}+\overline{u_j\sigma_{ik}})]\\ =\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-\left(\rho\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}+\rho\overline{u_ju_k}\frac{\partial U_i}{\partial x_k}+\rho\overline{u_iu_j}\frac{\partial U_k}{\partial x_k}\right)\\ +(\overline{u_if_j}+\overline{u_jf_i})-\overline{\left(\sigma_{ik}\frac{\partial u_j}{\partial x_k}+\sigma_{jk}\frac{\partial u_i}{\partial x_k}\right)},\end{aligned} \tag{3}$$

where p, σ_{ij} and f_i are the pressure, viscous-stress tensor and random body-force fluctuations. An overbar indicates an ensemble average. The terms in (3) are usually interpreted as the total rate of change of the Reynolds stress, the turbulent and viscous *diffusion*, *redistribution* of the turbulent energy among its different components, *production* due to the interaction between the mean flow and the Reynolds stress, *production* due to body forces, and viscous *dissipation* of the turbulent energy. Since we treat the simplest case of homogeneous isotropic turbulence, we introduce random body forces f_i as the driving mechanisms for the undisturbed turbulence. In a typical application, the turbulence might actually be driven at large scales by interactions between the mean shear and the Reynolds stress. However, as a result of the energy cascade process, the smaller-scale turbulence may be approximated as isotropic as assumed in the present analysis. It may be shown for acoustic interactions that the diffusion terms are generally of secondary importance (Noir 1975). The pressure–strain correlation or redistribution terms and the dissipation terms of (3) will be modelled using semi-empirical arguments.

For locally isotropic flows, the dissipation terms may be expressed as

$$\frac{1}{\rho}\left(\overline{\sigma_{ik}\frac{\partial u_j}{\partial x_k}}+\overline{\sigma_{jk}\frac{\partial u_i}{\partial x_k}}\right) = \tfrac{2}{3}\epsilon\,\delta_{ij}, \tag{4}$$

where ϵ is the rate of viscous dissipation. In equilibrium, the rate of viscous dissipation corresponds to the rate at which energy is transferred from large-scale eddies to small-scale eddies; it is independent of viscosity for high Reynolds number turbulence. We are interested in the amount of energy which is transformed by this nonlinear cascade process into viscous dissipation. Therefore it is reasonable to model the dissipation in terms of the large-scale end of the cascade. Daly & Harlow (1970) considered the dissipation to be proportional to the Reynolds stress. Using the same idea, we write

$$\overline{\sigma_{ik}\frac{\partial u_j}{\partial x_k}}+\overline{\sigma_{jk}\frac{\partial u_i}{\partial x_k}} = \frac{1}{\phi}\rho\overline{u_i u_j}\,\delta_{ij}, \tag{5}$$

where ϕ is a characteristic time for the decay of the turbulence.

For the pressure–strain correlation or redistribution terms, we first consider the simplest form of the model originally suggested by Rotta (1951). The basic idea is that the net energy exchange among components is proportional to the degree of anisotropy. Therefore a relaxation model for the redistribution is written as

$$\left.\overline{p_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}\right|_{\text{relax}} = -\frac{1}{\psi}(\rho\overline{u_i u_j}-\rho\overline{u_{\text{iso}}^2}\,\delta_{ij}), \tag{6}$$

where $\overline{u_{\text{iso}}^2} = \frac{1}{3}\overline{u_k u_k}$ and ψ has the dimensions of time and may be taken as a characteristic time for the return of the Reynolds stress to isotropy.

We now compare this simple relaxation model of the Reynolds-stress equation with the exact results which can be derived for an 'instantaneous' distortion of a turbulent field. An 'instantaneous' distortion is one which is rapid enough that the nonlinear redistribution and cascade terms do not have time to act significantly, i.e. distortion occurring over times short compared with ϕ and ψ. In such a case, the redistribution and dissipation models given above will not contribute to the Reynolds-stress equation. Only the mean-flow production terms will contribute, giving for our case

$$d(\rho\overline{u_i u_j})/dt = -B_{ij}\rho_0\overline{u_{\text{iso}}^2}\,\partial U_1/\partial x_1 + O(\delta^2), \tag{7}$$

where

$$\mathbf{B} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

The effect of an 'instantaneous' distortion on the behaviour of the longitudinal and lateral components of the turbulent energy is given exactly by Batchelor (1953, p. 68) for an incompressible mean flow and by Ribner & Tucker (1952) for a compressible mean flow. Both these analyses consider the turbulence to be initially isotropic and the distortion to be uniform. Using the last reference, the change in the Reynolds stress for our case is computed to be (see appendix)

$$d(\overline{\rho u_i u_j})/dt = -A_{ij}\rho_0 \overline{u^2_{\text{iso}}}\,\partial U_1/\partial x_1 + O(\delta^2), \tag{9}$$

where

$$\mathbf{A} = \begin{pmatrix} \frac{11}{5} & 0 & 0 \\ 0 & \frac{7}{5} & 0 \\ 0 & 0 & \frac{7}{5} \end{pmatrix}. \tag{10}$$

This result differs from (8) but, since $A_{ii} = B_{ii} = 5$, it is clear that both methods give the same result for the total turbulent energy.

However, (6) does not account for the rapid redistribution of energy among components. This mean-strain, or rapid-redistribution, effect was modelled by Rotta (1951) for isotropic turbulence as

$$\left.\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}\right|_{\text{rapid}} = \tfrac{3}{5}\rho_0\overline{u^2_{\text{iso}}}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right),$$

or for our case as

$$\left.\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}\right|_{\text{rapid}} = C_{ij}\rho_0\overline{u^2_{\text{iso}}}\frac{\partial U_1}{\partial x_1} + O(\delta^2), \tag{11}$$

where the coefficients would be

$$\mathbf{C} = \begin{pmatrix} \frac{6}{5} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Adding (11) with these coefficients to the Reynolds-stress equation is still not sufficient to match the exact results (8) and (10) of Ribner & Tucker. In order to do so, we introduce a modification such that **C** in the rapid part is taken as

$$\mathbf{C} = \begin{pmatrix} \frac{6}{5} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} \frac{2}{5} & 0 & 0 \\ 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{2}{5} \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & 0 & 0 \\ 0 & -\frac{2}{5} & 0 \\ 0 & 0 & -\frac{2}{5} \end{pmatrix}. \tag{12}$$

It may be noted that now, since $C_{ii} = 0$, this term does not change the overall turbulent energy during a rapid distortion.

For our case of a one-dimensional wave, the Reynolds-stress equation (3) with (4)–(12) then becomes

$$\frac{d}{dt}\overline{\rho u_i u_j} = -\frac{1}{\psi}(\overline{\rho u_i u_j} - \overline{\rho u^2_{\text{iso}}}\,\delta_{ij}) - \frac{1}{\phi}\overline{\rho u_i u_j}\,\delta_{ij} + \overline{u_i f_j} + \overline{u_j f_i} - A_{ij}\rho_0\overline{u^2_{\text{iso}}}\frac{\partial U_1}{\partial x_1} + O(\delta^2). \tag{13}$$

Now if the acoustic wavelength is large compared with the turbulent scales the distortion due to the wave will be locally homogeneous and the Reynolds stresses may be considered as functions of time or the acoustic wave phase only. As the whole process does not involve any shear, the Reynolds shear stress, which was zero before the arrival of the wave (homogeneous turbulence), is still zero. If the normal stresses are written as

$$m = \rho_0 \overline{u_1 u_1}, \quad n = \rho_0 \overline{u_2 u_2}, \quad p = \rho_0 \overline{u_3 u_3},$$

(13) may be written as three scalar equations which in canonical form read

$$\dot{m} + \frac{1}{3\psi}(2m-n-p) + \frac{1}{\phi}m - 2\overline{u_1 f_1} = \frac{11}{5}\frac{\omega\delta}{\gamma} m \sin(kx_1 - \omega t) + O(\delta^2), \tag{14a}$$

$$\dot{n} + \frac{1}{3\psi}(2n-m-p) + \frac{1}{\phi}n - 2\overline{u_2 f_2} = \frac{7}{5}\frac{\omega\delta}{\gamma} n \sin(kx_1 - \omega t) + O(\delta^2), \tag{14b}$$

$$\dot{p} + \frac{1}{3\psi}(2p-m-n) + \frac{1}{\phi}p - 2\overline{u_3 f_3} = \frac{7}{5}\frac{\omega\delta}{\gamma} p \sin(kx_1 - \omega t) + O(\delta^2). \tag{14c}$$

The corresponding reference conditions (no wave) are

$$m = m_0, \quad n = n_0, \quad p = p_0 \quad \text{for} \quad t \leqslant 0. \tag{15}$$

Equations (14) are similar to the equation of an electric parallel RC circuit and to the equation of the viscoelastic Maxwell body (a spring and a dashpot in series) in continuum mechanics. The viscoelastic character of turbulent flows, first pointed out by Rivlin (1957), has been broadly discussed since. However, from (14), it is obvious that the so-called viscoelastic properties of the Reynolds stress are due to the redistribution processes (among components or in wavenumber space); they are independent of viscosity, except at very small scales. To avoid possible confusion, the term *inelastic* is preferable to viscoelastic.

Neglecting the transient associated with the initiation of the interaction, the solution of (14) is assumed to be of the form

$$m = m_0[1 + \delta a_1 \cos(kx_1 - \omega t) + \delta a_2 \sin(kx_1 - \omega t)] + O(\delta^2), \tag{16a}$$

$$n = n_0[1 + \delta b_1 \cos(kx_1 - \omega t) + \delta b_2 \sin(kx_1 - \omega t)] + O(\delta^2), \tag{16b}$$

$$p = p_0[1 + \delta c_1 \cos(kx_1 - \omega t) + \delta c_2 \sin(kx_1 - \omega t)] + O(\delta^2). \tag{16c}$$

The zeroth-order solution leads to

$$(3\psi)^{-1}(2m_0 - n_0 - p_0) + \phi^{-1} m_0 = 2\overline{u_1 f_1}, \tag{17a}$$

$$(3\psi)^{-1}(2n_0 - m_0 - p_0) + \phi^{-1} n_0 = 2\overline{u_2 f_2}, \tag{17b}$$

$$(3\psi)^{-1}(2p_0 - m_0 - n_0) + \phi^{-1} p_0 = 2\overline{u_3 f_3}. \tag{17c}$$

This equation expresses the production–dissipation balance, i.e. shows how random body-force production balances redistribution and dissipation.

For reasons of simplicity, the coefficients a_1, b_1, etc. are computed explicitly in the case of initially isotropic turbulence only ($m_0 = n_0 = p_0$). Another useful but inessential simplification results if we assume high frequency sound so that

$$\omega\psi \gg 1, \quad \omega\phi \gg 1. \tag{18), (19}$$

Then the second-order quantities of the form $(\omega^2\psi\phi)^{-1}$ may be dropped. This assumption is valid for most cases of practical interest and is approximately valid for the self-generated sound from turbulence.

For the first-order terms identification of the sine and cosine terms leads to four equations (by symmetry, $n = p$, $b_1 = c_1$ and $b_2 = c_2$). The coefficients are found to be

$$a_1 = \frac{11}{5\gamma}, \quad b_1 = c_1 = \frac{7}{5\gamma}, \tag{20a, b}$$

$$a_2 = \frac{8}{15\gamma}\frac{1}{\omega\psi} + \frac{11}{5\gamma}\frac{1}{\omega\phi}, \tag{20c}$$

$$b_2 = c_2 = -\frac{4}{15\gamma}\frac{1}{\omega\psi} + \frac{7}{5\gamma}\frac{1}{\omega\phi}. \tag{20d, e}$$

3. The turbulent kinetic energy equation

The contraction of the Reynolds-stress equation (3) gives the turbulent kinetic energy equation

$$\frac{dE_t}{dt} + \frac{\partial}{\partial x_k}(\tfrac{1}{2}\overline{\rho u_i u_i u_k} + \overline{pu_k} - \overline{u_i \sigma_{ik}}) = -\overline{\rho u_i u_k}\frac{\partial U_i}{\partial x_k} - \tfrac{1}{2}\overline{\rho u_i u_i}\frac{\partial U_k}{\partial x_k} + \overline{p\frac{\partial u_i}{\partial x_i}} + \overline{u_i f_i} - \overline{\sigma_{ik}\frac{\partial u_i}{\partial x_{ik}}}. \tag{21}$$

The absorption of acoustic energy by turbulence is equal to (minus) the part of the production of turbulent kinetic energy due to the wave. The word 'production' now has to be interpreted in a broad sense since, when dealing with a compressible mean flow, the contracted pressure–strain correlation term could play the role of an energy source and does not necessarily vanish. However, in our case using (6) and (11) to model the redistribution shows that the term does vanish. The production then reads

$$\left.\frac{dE_t}{dt}\right|_{\text{prod}} = -\overline{\rho u_1 u_1}\frac{\partial U_1}{\partial x_1} - \tfrac{1}{2}\overline{\rho u_i u_i}\frac{\partial U_1}{\partial x_1}. \tag{22}$$

The first part describes production due to the work done by the Reynolds stress against the wave, the second part that due to compressibility effects. The average production is obtained by taking the time average of the production over one acoustic period; using in (22) the expressions (16) and (20) for the time-varying Reynolds stress, this is given by

$$\left\langle \left.\frac{dE_t}{dt}\right|_{\text{prod}} \right\rangle = \frac{1}{T}\int_{-\frac{1}{2}T}^{\frac{1}{2}T} \frac{1}{\gamma}\omega\delta^2 \sin^2(kx_1 - \omega t)\, m_0\{a_2 + \tfrac{1}{2}(a_2 + b_2 + c_2)\}\, dt = m_0\frac{\delta^2}{\gamma^2}\left(\frac{4}{15}\frac{1}{\psi} + \frac{47}{20}\frac{1}{\phi}\right). \tag{23}$$

The production is positive, so that the acoustic energy is seen to decrease during the interaction.

The attenuation coefficient corresponding to the absorption of sound is derived from the definition

$$\alpha_t = dQ/I_0, \tag{24}$$

where dQ is the acoustic energy loss per unit volume and $I_0 = \rho_0 a_0 |U_0|^2$ is the intensity of the incoming wave. As $|U_0| = a_0\delta/\gamma$,

$$\alpha_t = (\overline{u_1^2})_0 \frac{1}{a_0^3}\left(\frac{4}{15}\frac{1}{\psi} + \frac{47}{20}\frac{1}{\phi}\right). \tag{25}$$

In his analysis, Crow (1967) gets a related expression, with the argument of his 'memory function' instead of ϕ^{-1}. On the basis of existing experimental results, Lumley (1970) found that the return to isotropy is faster than the decay by a factor of the order of 4, so that

$$\psi \simeq \tfrac{1}{4}\phi. \tag{26}$$

Comparison of (5) and (6) shows that for locally isotropic turbulence

$$\phi \simeq \tfrac{3}{2}\overline{u_{\mathrm{iso}}^2}/\epsilon. \tag{27}$$

Therefore the attenuation coefficient becomes

$$\alpha_t \simeq 2{\cdot}3\epsilon/a_0^3. \tag{28}$$

4. Assumptions

The present analysis is based on several assumptions; some are quite reasonable, others are weaker. The first basic assumption is concerned with the size of the parameter δ characterizing the wave amplitude; it has to be small enough to ensure the convergence of the above expansions. As δ is generally of order 10^{-3} for the loudest sounds of interest, this assumption is not restrictive.

As previously mentioned, the neglect of diffusion terms in the Reynolds-stress equation would not be expected to affect the first-order results.

In order to simplify the algebra we restricted some developments to high frequency waves: $\omega\psi \gg 1$. This condition is not always fulfilled, for example in the case of the propagation of very low frequency waves through the atmosphere. However, when this condition is not strictly satisfied, the energy transfer mechanisms are similar and the order of magnitude of the present results should still be useful.

Turbulence modelling is generally based on experimental results; here the quantities ϕ and ψ are not known, and are estimated on the basis of physical reasoning in the applications. This is certainly the major source of errors in the applications.

An important assumption was concerned with the locally homogeneous character of the distortion. This assumption may be removed if we assume that only eddies of scale smaller than the acoustic wavelength contribute to the attenuation process. This assumption seems reasonable, since, when the acoustic wavelength is smaller than the scale of a given turbulent Fourier component, the space-average distortion of that turbulent component will be negligible. From spectral analysis, it may be shown that the contribution to the absorption process of eddies of wavenumber κ in the range $[\kappa, \kappa + d\kappa]$ is approximately given by

$$\alpha_t(\kappa) \simeq 0{\cdot}7a_0^{-3}[E(\kappa)\,\kappa]^{\frac{3}{2}}\,d\kappa. \tag{29}$$

For turbulence which can be characterized very roughly by a spectrum of the form

$$E(\kappa) \simeq 1{\cdot}5\epsilon^{\frac{2}{3}}\kappa^{-\frac{5}{3}}, \tag{30}$$

i.e. by only an inertial range extending from L_0, the largest length scale of the flow, to the Kolmogorov scale η, the attenuation coefficient can be found to be

$$\alpha_t \simeq 1{\cdot}3a_0^{-3}\epsilon \ln(L_0/\eta). \tag{31}$$

In the case where the acoustic wavelength is smaller than the integral scale, the lower limit of integration (over wavenumbers) is taken as the acoustic wavenumber, so that

$$\alpha_t \simeq 1\cdot 3a_0^{-3}\epsilon \ln(\lambda_w/\eta). \tag{32}$$

5. Comparison between theory and *experiment*

Turbulence generally occurs in conjunction with mean flows such as wind, buoyancy-induced flows, jets, wakes, boundary layers, etc. Therefore measurements of sound–turbulence interactions include refraction, scattering and absorption by turbulence, as well as molecular and viscous effects. Molecular and viscous effects are well understood and are usually subtracted from the experimental data. Refraction by temperature and velocity gradients may be taken into account only if the gradients are described by simple functions and geometrical acoustics applies; refraction by large-scale turbulent motions is difficult to analyse. As a consequence, comparison of the results of the present analysis with measurements of atmospheric attenuation attributed to turbulence showed strong discrepancies, suggesting that the turbulent absorption is of only secondary importance in that case; the strong effect of turbulence on sound attenuation then must be explained by other mechanisms, such as scattering and ground absorption.

The experiment of Hunter & Lowson

In order to measure the attenuation of a sound wave due to turbulence effects only, Hunter & Lowson (1974) devised the following experiment: they measured the reverberation time of a small room, first under quiescent conditions, then while the air of the room was strongly turbulent. The reverberation time was found to be shorter for turbulent conditions, corresponding to an extra attenuation coefficient of order $10^{-3}\,\mathrm{m}^{-1}$ attributable to turbulence interactions. The measured attenuation coefficient was approximately independent of frequency, whereas scattering and spectral broadening are proportional to frequency squared. In addition to energy absorbed by the turbulence, there may be three other effects: changes in the acoustic propagation speed, scattering of the waves and spectral broadening. As these other interactions do not involve a change in the energy balance, Hunter & Lowson suggested that acoustic energy may be absorbed by the turbulent field.

Another conceivable explanation might be that acoustic energy is absorbed by the boundary layers on the moving fan blades. This mechanism is related to the experimental and theoretical results (Ingard & Singhal 1974) that sound propagating along a duct is attenuated owing to the quasi-steady change in turbulent friction due to the velocity fluctuations of the sound. It might seem that the fan-blade boundary layers would behave similarly and explain the measured additional attenuation. A crude order-of-magnitude estimate of this effect was made by assuming that the blades' absorption coefficient per unit area is the same as that of the duct walls in Ingard & Singhal's results. Using a blade area of twice the fan-disk area, one finds that the equivalent attenuation coefficient for this effect is of order $6 \times 10^{-6}\,\mathrm{m}^{-1}$, which is far smaller than the measured additional attenuation.

To compare theory with experiment, a detailed description of the turbulent flow field is required. Unfortunately, only one hot-wire probe was used. As a rough approximation, we may say that the flow in the room is jet-like downstream of the fan and

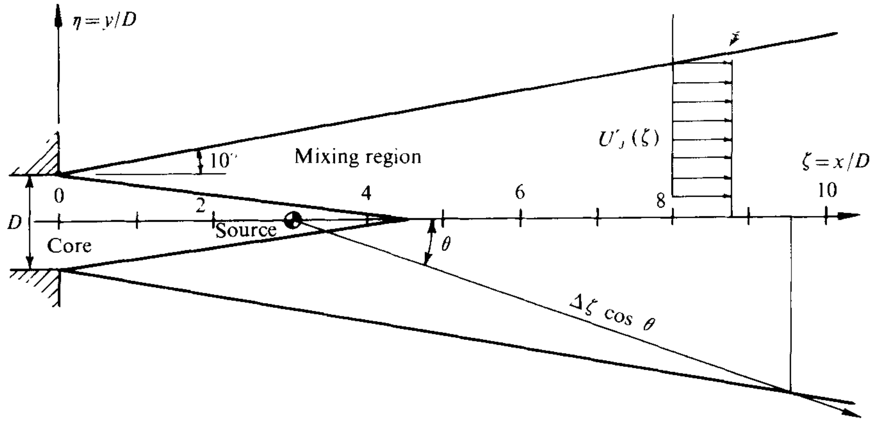

FIGURE 1. Sketch of a subsonic round jet.

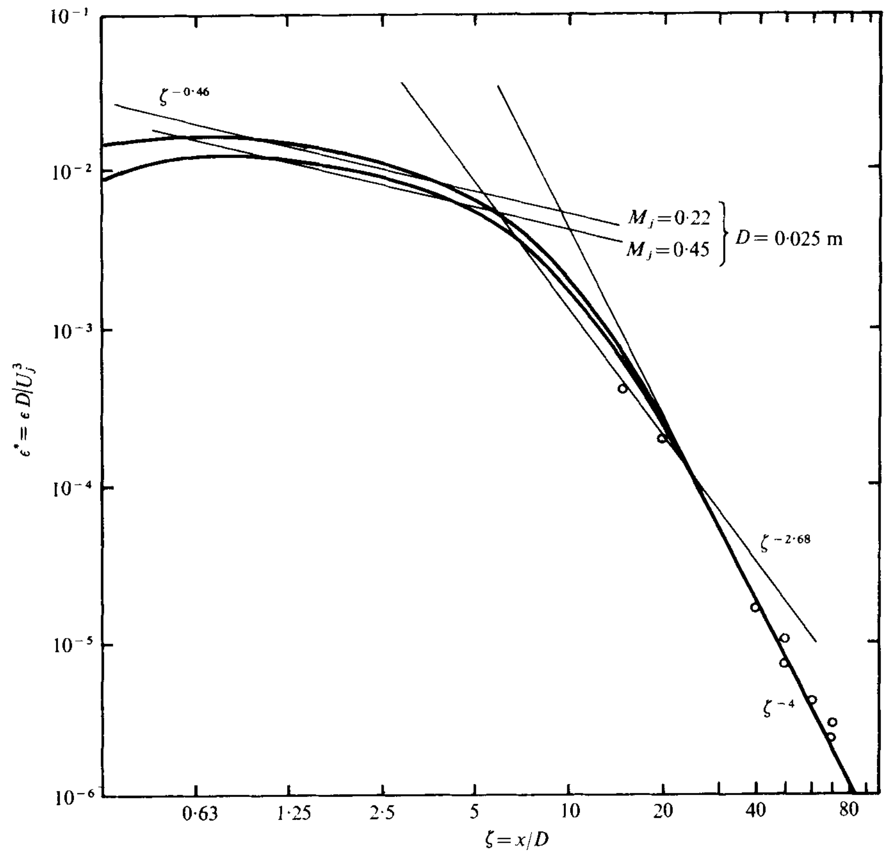

FIGURE 2. Dimensionless dissipation rate as a function of position. ○, experiment (Friehe *et al.* 1972).

sink-like upstream. This implies that the turbulent flow pattern inside the room is quite heterogeneous and leads to large differences between methods of estimating the rate of viscous dissipation ϵ in the room. In addition, the present analysis assumes homogeneous turbulence. According to the hot-wire measurements,

$$\epsilon \sim (\overline{u^2})^{\frac{3}{2}}/l \sim 10\,\mathrm{m^2\,s^{-3}}.$$

An overall production–dissipation balance based on the energy input of the fan leads to $\epsilon \sim 100\,\mathrm{m^2\,s^{-3}}$ whilst maximum specific kinetic energy input arguments give $\epsilon \sim 1000\,\mathrm{m^2\,s^{-3}}$. Using the crude estimate $\epsilon = 100$–$1000\,\mathrm{m^2\,s^{-3}}$, the predicted attenuation coefficient becomes 10^{-5}–$10^{-4}\,\mathrm{m^{-1}}$, while the measured attenuation is of order $10^{-3}\,\mathrm{m^{-1}}$. However, as noted by Hunter & Lowson, the experimental results are only approximate and nearly qualitative.

The experiment of Lush

The measurements of subsonic jet noise by Lush (1971) were made for a particularly clean configuration. Comparison with Lighthill's theory, which does not take refraction into account, shows some discrepancies, particularly for small angles with respect to the jet axis and at high frequencies. For example, Lighthill's quadrupole analogy predicts that the frequency of the emitted sound increases with increasing jet speed; experiment shows that this is not true close to the jet axis. Lush also measured a breakdown of the convective amplification effect near the jet axis. Several of these discrepancies may be explained by refraction and scattering arguments as well as by considering the turbulent absorption of sound. However, as will be seen below, the discrepancies occurring in the total acoustic power of the jet cannot be explained by refraction and scattering, and may be at least partly attributed to turbulent absorption, though perhaps other explanations could also be given.

A schematic sketch of the jet is given in figure 1. Because of the self-preserving character of the flow, it is convenient to introduce the dimensionless variables

$$\zeta = x/D \quad \text{and} \quad \eta = y/D \quad (D = 0{\cdot}025\ \mathrm{m}\ \text{in the experiment of Lush}).$$

For simplicity, the different flow properties are considered to be constant in a plane normal to the jet axis, though they actually vary strongly in the shear region.

For homogeneous, nearly isotropic turbulence, the theory predicts that the absorption is proportional to the viscous dissipation rate ϵ. The longitudinal distribution of the dimensionless dissipation rate is given by Friehe, Van Atta & Gibson (1972) for a fully developed jet; it has a universal character with slope ζ^{-4}, as shown in figure 2. The curve is extended to the mixing region on the basis of Rotta's (1951) relationship for locally isotropic turbulence at high Reynolds number:

$$\epsilon \simeq 0{\cdot}2(\tfrac{3}{2}\overline{u^2})^{\frac{3}{2}}/l, \tag{33}$$

where l is an integral scale. The values of l and the turbulence intensity $(\overline{u^2})^{\frac{1}{2}}$ as a function of the longitudinal co-ordinate are taken from Davies, Fisher & Barratt (1963). The value of the dimensionless dissipation rate $\epsilon^* = \epsilon D U_j^{-3}$ is plotted in figure 2. In the mixing region, it is a function of position and Mach number. For further convenience, the two curves are approximated by straight lines with slope $\zeta^{-0{\cdot}46}$ and $\zeta^{-2{\cdot}68}$.

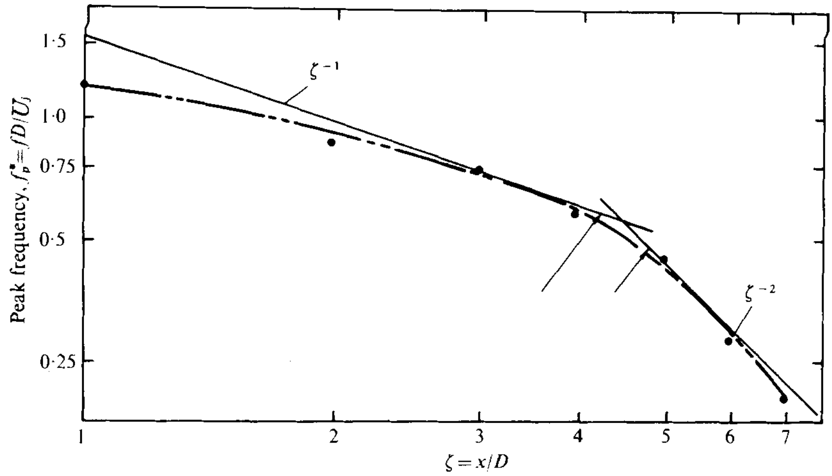

FIGURE 3. Dimensionless peak frequency as a function of position. ●, experiment (Lee 1971).

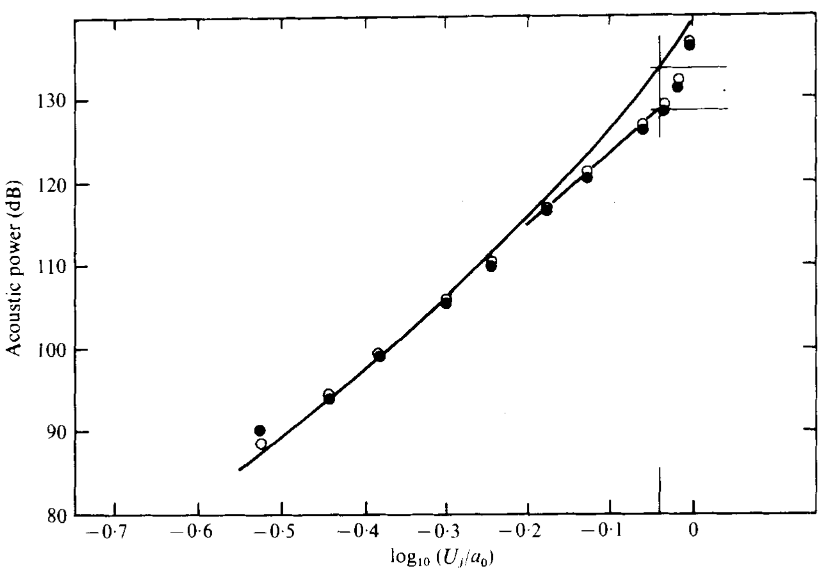

FIGURE 4. Velocity dependence of acoustic power compared with theory. ○, ●, experiment (Lush 1971).

Using dimensional analysis, Powell (1953) estimated the longitudinal distribution of the source peak frequencies as well as the noise spectrum. The frequency is proportional to ζ^{-1} in the mixing region and to ζ^{-2} in the fully developed jet. These theoretical results have been confirmed experimentally; figure 3 shows typical results due to Lee (1971).

The basic sound field of a jet is roughly ellipsoidal as discussed by Ribner (1969) but convection of the quadrupoles by the mean flow results in a strong amplification in the direction of the flow. In addition, velocity gradients refract acoustic rays out of the jet, making the actual noise pattern heart-shaped. The discrepancies between the measurements of the acoustic field, which show the above effects, and the predictions of Lighthill, which do not account for refraction, might be due to refraction (Schubert 1972);

however, as mentioned previously, sound scattering and refraction do not change the overall energy balance. Therefore the discrepancy between measurements and predictions of the total acoustic *power* of the jet (figure 4) cannot be explained by refraction and scattering effects, and may be associated with turbulent absorption.

From figures 2 and 3, it is evident that a high frequency wave, emitted near the nozzle exit and propagating along the jet axis, travels through a region of relatively high dissipation rate and may be significantly damped; if it propagates in a direction normal to the jet through only a thin layer of turbulent flow, its damping is negligible. Also a lower frequency wave, emitted downstream of the potential core, is not damped significantly, since it only travels through weaker turbulence (in any direction).

An order-of-magnitude estimate of this phenomenon has been carried out (Noir 1975). The effective location of the acoustic sources is assumed to be $\zeta = 3$ (figure 1). The intensity loss in the turbulent field along a given ray is computed from (28) and then integrated over a spherical shell of radius $R = 3$ m, as for the experiments. For a jet Mach number equal to 0·9, the total acoustic energy absorbed is found to be of order 0·1 W. Lush (1971) measured a total acoustic power of 0·6 W while Lighthill's theory as applied by Lush predicts 2 W, giving a discrepancy in energy of 1·4 W. It may be concluded that, in view of all the above approximations, the present analysis of turbulent absorption gives a rough estimate of at least part of the energy defect.

6. Conclusions

The Reynolds-stress equations and the turbulent kinetic energy equation were used to determine the production of turbulent energy due to an acoustic wave or in other words, the absorption of acoustic energy by a turbulent field. In the Reynolds-stress equations, the redistribution terms (among components and among wavenumbers) were replaced by simple models. In the present analysis the constants appearing in these models have not been determined experimentally, so that order-of-magnitude estimates were introduced on the basis of physical mechanisms and comparison with known theoretical results. Owing to the opposing effects of the distortion of the turbulence (producing Reynolds stress) and of the redistribution of the kinetic energy among components (tendency towards isotropy) and among wavenumbers (energy cascade), it is seen that the Reynolds stress behaves similarly to the stress in an inelastic body. As a result it is not in phase with the acoustic strain field (§2), and there is an average production of turbulent energy and thus attenuation of the wave.

The analysis applies to one-dimensional sound waves in homogeneous isotropic turbulence and could be extended to sound propagation in simply sheared homogeneous anisotropic turbulence. Some assumptions were introduced to simplify the algebra, but they were not of fundamental importance and can be removed. The assumption of uniform distortion was removed by including in the computation only those eddies which are smaller than the acoustic wavelength; such reasoning remains to be verified.

Comparison with existing experimental results shows that the results of the present analysis are of nearly the correct order of magnitude but generally low; this suggests that the theoretical as well as the experimental approach should be refined. From the theoretical point of view, a more refined model of the Reynolds-stress equation could be introduced, better estimates for the characteristic times ϕ and ψ could be found, etc.

From the experimental point of view, experiments satisfying the assumptions of the analysis should be carried out (homogeneous turbulence, simple turbulent shear flow, etc.). The approach of Hunter & Lowson is particularly fruitful in excluding most non-turbulent effects, but the character of the turbulent flow in their particular experiment was quite complicated and difficult to compare with the analysis. In atmospheric attenuation measurements, it is suggested that the frequency dependence is a useful parameter with which to compare attenuation mechanisms. If radial-spreading, molecular and relaxation effects are subtracted from the measurements, then *scattering* increases with frequency, *refraction* is negligible at very low frequencies and becomes more important at high frequencies (geometrical acoustics), while the present phenomenon of *turbulent absorption* is essentially frequency independent (except at very low frequencies). In any future experiments, it will be necessary to focus attention on both acoustic and turbulent measurements.

In summary, this paper analyses a mechanism for the absorption of acoustic energy by turbulence. The dynamics of the response of the turbulence to distortion were modelled and the magnitude of the relaxation time scales estimated by an order-of-magnitude approach. Agreement with experiment is not yet very good from a quantitative point of view. When both turbulent flow and noise are present, the phenomenon may be of importance in aeroacoustics and noise-control technology.

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Appendix. Computation of the instantaneous Reynolds-stress production after Ribner & Tucker

In the case of a wind-tunnel contraction, a cubic element $D \times D \times D$ is deformed into a parallelepiped $Dl_1 \times Dl_2 \times Dl_3$; l_1, l_2 and l_3 are the normal strain components. The continuity condition requires

$$\sigma l_1 l_2 l_3 = 1,$$

where $\sigma = \rho/\rho_0$. The distortion caused by an acoustic wave is comparable to a fictitious contraction with $l_2 = l_3 = 1$ and $l_1 = 1/\sigma = \rho_0/\rho$. Then Ribner & Tucker's method may be used to describe the effect of an instantaneous acoustic distortion on the Reynolds stresses. If the turbulence is homogeneous and initially isotropic, the longitudinal and lateral correlations are given by Ribner & Tucker (1952) as

$$\left.\begin{aligned} \frac{\overline{u_1 u_1}}{u_{\text{iso}}^2} &= \frac{3}{4l_1^2}\left[\frac{-1}{1-l_1^{-2}} + \frac{2-l_1^{-2}}{(1-l_1^{-2})^{\frac{3}{2}}}\tanh^{-1}(1-l_1^{-2})^{\frac{1}{2}}\right], \\ \frac{\overline{u_2 u_2}}{u_{\text{iso}}^2} = \frac{\overline{u_3 u_3}}{u_{\text{iso}}^2} &= \frac{3}{8}\left[\frac{2-l_1^{-2}}{1-l_1^{-2}} - \frac{l_1^{-4}}{(1-l_1^{-2})^{\frac{3}{2}}}\tanh^{-1}(1-l_1^{-2})^{\frac{1}{2}}\right], \end{aligned}\right\} \quad (\text{A } 1)$$

with

$$l_1 = \frac{\rho_0}{\rho} = \frac{1}{1+(\delta/\gamma)\cos(kx_1-\omega t)+O(\delta^2)} = 1-(\delta/\gamma)\cos(kx_1-\omega t)+O(\delta^2), \quad (\text{A } 2)$$

where ρ is computed from (1).

These correlations become, to first order,

$$\left.\begin{aligned}\frac{\overline{u_1 u_1}}{\overline{u^2_{1\text{so}}}} &= 1+\frac{6}{5}\frac{U_1}{a}+O(\delta^2),\\ \frac{\overline{u_2 u_2}}{\overline{u^2_{\text{iso}}}} = \frac{\overline{u_3 u_3}}{\overline{u^2_{\text{iso}}}} &= 1+\frac{2}{5}\frac{U_1}{a}+O(\delta^2),\end{aligned}\right\} \qquad \text{(A 3)}$$

or

$$\left.\begin{aligned}\overline{\rho u_1 u_1} &= \rho_0 \overline{u^2_{\text{iso}}}\left[1+\frac{11}{5}\frac{U_1}{a}+O(\delta^2)\right],\\ \overline{\rho u_2 u_2} = \overline{\rho u_3 u_3} &= \rho_0 \overline{u^2_{\text{iso}}}\left[1+\frac{7}{5}\frac{U_1}{a}+O(\delta^2)\right]\end{aligned}\right\} \qquad \text{(A 4)}$$

if density acoustic fluctuations are accounted for. The instantaneous production of Reynolds stress is obtained by differentiating (A 4) with respect to time:

$$\frac{d(\overline{\rho u_i u_j})}{dt} = -\rho_0 \overline{u^2_{\text{iso}}}\frac{\partial U_1}{\partial x_1}A_{ij}+O(\delta^2), \qquad \text{(A 5)}$$

where

$$\mathbf{A} = \begin{pmatrix}\frac{11}{5} & 0 & 0\\ 0 & \frac{7}{5} & 0\\ 0 & 0 & \frac{7}{5}\end{pmatrix}.$$

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